# GCSE Mathematics (1MA1) FOUNDATION TIER 

For first teaching from September 2015

| Unit | Title | Estimated hours |
| :---: | :---: | :---: |
| 1 | Integers and place value | 6 |
|  | Decimals | 5 |
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|  | Factors, multiples and primes | 6 |
| a | Algebra: the basics | 8 |
| $\underline{2} \quad \underline{b}$ | Expanding and factorising single brackets | 6 |
| C | Expressions and substitution into formulae | 7 |
| $\underline{3}$ | Tables | 7 |
|  | Charts and graphs | 7 |
|  | Pie charts | 4 |
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| $4 \quad \underline{b}$ | Fractions, decimals and percentages | 4 |
| c | Percentages | 7 |
| $\underline{\text { a }}$ | Equations | 7 |
| $\underline{5} \quad \underline{b}$ | Inequalities | 5 |
| C | Sequences | 7 |
| 6 a | Properties of shapes, parallel lines and angle facts | 10 |
| $\underline{6} \quad \underline{b}$ | Interior and exterior angles of polygons | 6 |
| 7 a | Statistics and sampling | 4 |
| $\underline{7}$ | The averages | 6 |
| 8 a | Perimeter and area | 10 |
| $\underline{8} \quad \underline{b}$ | 3D forms and volume | 6 |
| 9 a | Real-life graphs | 9 |
| $\underline{9} \quad \underline{b}$ | Straight-line graphs | 6 |
| 10 a | Transformations I: translations, rotations and reflections | 6 |
| $\underline{10}$ | Transformations II: enlargements and combinations | 8 |
| 11 a | Ratio | 6 |
| 11 b | Proportion | 6 |
| 12 | Right-angled triangles: Pythagoras and trigonometry | 6 |
| 13 a | Probability I | 5 |
| $\underline{13}$ | Probability II | 9 |
| 14 | Multiplicative reasoning | 7 |
| 15 a | Plans and elevations | 6 |
| $\underline{15}$ b | Constructions, loci and bearings | 10 |
| 16 | Quadratic equations: expanding and factorising | 5 |
|  | Quadratic equations: graphs | 4 |
| 17 | Circles, cylinders, cones and spheres | 7 |
| 18 | Fractions and reciprocals | 5 |
|  | Indices and standard form | 6 |
| $\underline{19}$ | Similarity and congruence in 2D | 7 |
|  | Vectors | 7 |
| $\underline{20}$ | Rearranging equations, graphs of cubic and reciprocal functions and simultaneous equations | 5 |

## UNIT 1: Number, powers, decimals, HCF and LCM, roots and rounding

## KEYWORDS

Integer, number, digit, negative, decimal, addition, subtraction, multiplication, division, remainder, operation, estimate, power, roots, factor, multiple, primes, square, cube, even, odd

## 1a. Integers and place value

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Use and order positive and negative numbers (integers);
- Order integers, decimals, use the symbols $<$, > and understand the $\neq$ symbol;
- Add and subtract positive and negative numbers (integers);
- Recall all multiplication facts to $10 \times 10$, and use them to derive quickly the corresponding division facts;
- Multiply or divide any number by powers of 10;
- Multiply and divide positive and negative numbers (integers);
- Use brackets and the hierarchy of operations (not including powers);
- Round numbers to a given power of 10;
- Check answers by rounding and using inverse operations.


## NOTES

Particular emphasis should be given to the importance of students presenting their work clearly. Formal written methods of addition, subtraction and multiplication work from right to left, whilst formal division works from left to right.
Any correct method of multiplication will still gain full marks, for example, the grid method, the traditional method, Napier's bones.
Negative numbers in real life can be modelled by interpreting scales on thermometers using $F$ and $C$.
Encourage the exploration of different calculation methods.
Students should be able to write numbers in words and from words as a real-life skill.

## 1b. Decimals

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Use decimal notation and place value;
- Identify the value of digits in a decimal or whole number;
- Compare and order decimal numbers using the symbols <, >;
- Understand the $\neq$ symbol (not equal);
- Write decimal numbers of millions, e.g. $2300000=2.3$ million;
- Add, subtract, multiply and divide decimals;
- Multiply or divide by any number between 0 and 1 ;
- Round to the nearest integer;
- Round to a given number of decimal places;
- Round to any given number of significant figures;
- Estimate answers to calculations by rounding numbers to 1 significant figure;
- Use one calculation to find the answer to another.


## NOTES

Practise long multiplication and division, use mental maths problems with decimals such as 0.1 , 0.001 .

## 1c. Indices, powers and roots

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Find squares and cubes:
- recall integer squares up to $10 \times 10$ and the corresponding square roots;
- understand the difference between positive and negative square roots;
- recall the cubes of $1,2,3,4,5$ and 10 ;
- Use index notation for squares and cubes;
- Recognise powers of 2, 3, 4, 5;
- Evaluate expressions involving squares, cubes and roots:
- add, subtract, multiply and divide numbers in index form;
- cancel to simplify a calculation;
- Use index notation for powers of 10, including negative powers;
- Use the laws of indices to multiply and divide numbers written in index notation;
- Use the square, cube and power keys on a calculator;
- Use brackets and the hierarchy of operations with powers inside the brackets, or raising brackets to powers;
- Use calculators for all calculations: positive and negative numbers, brackets, powers and roots, four operations.


## NOTES

Pupils need to know how to enter negative numbers into their calculator.
Use the language of 'negative' number and not minus number to avoid confusion with calculations.
Note that the students need to understand the term 'surd' as there will be occasions when their calculator displays an answer in surd form, for example, $4 \sqrt{ } 2$.

## 1d. Factors, multiples and primes

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- List all three-digit numbers that can be made from three given integers;
- Recognise odd and even numbers;
- Identify factors, multiples and prime numbers;
- Recognise two-digit prime numbers;
- List all factors of a number and list multiples systematically;
- Find the prime factor decomposition of positive integers and write as a product using index notation;
- Find common factors and common multiples of two numbers;
- Find the LCM and HCF of two numbers, by listing, Venn diagrams and using prime factors: include finding LCM and HCF given the prime factorisation of two numbers;
- Understand that the prime factor decomposition of a positive integer is unique - whichever factor pair you start with - and that every number can be written as a product of two factors;
- Solve simple problems using HCF, LCM and prime numbers.


## NOTES

Use a number square to find primes (Eratosthenes sieve).
Using a calculator to check factors of large numbers can be useful.
Students need to be encouraged to learn squares from $2 \times 2$ to $15 \times 15$ and cubes of 2, 3, 4, 5 and 10 and corresponding square and cube roots

UNIT 2: Expressions, substituting into simple formulae, expanding and factorising

## KEYWORDS

Expression, identity, equation, formula, substitute, term, 'like' terms, index, power, collect, substitute, expand, bracket, factor, factorise, linear, simplify

## 2a. Algebra: the basics

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Use notation and symbols correctly;
- Write an expression;
- Select an expression/equation/formula/identity from a list;
- Manipulate and simplify algebraic expressions by collecting 'like' terms;
- Multiply together two simple algebraic expressions, e.g. $2 a \times 3 b$;
- Simplify expressions by cancelling, e.g. $\frac{4 x}{2}=2 x$;
- Use index notation when multiplying or dividing algebraic terms;
- Use index laws in algebra;
- Use index notation in algebra.
- Understand the $\neq$ symbol and introduce the identity $\equiv$ sign;


## NOTES

Some of this will be a reminder from Key Stage 3.
Emphasise correct use of symbolic notation, i.e. $3 \times y=3 y$ and not $y 3$ and $a \times b=a b$. Use lots of concrete examples when writing expressions, e.g. 'B' boys + 'G' girls.
Plenty of practice should be given and reinforce the message that making mistakes with negatives and times tables is a different skill to that being developed.

## 2b. Expanding and factorising single brackets

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Multiply a single number term over a bracket;
- Write and simplify expressions using squares and cubes;
- Simplify expressions involving brackets, i.e. expand the brackets, then add/subtract;
- Argue mathematically to show algebraic expressions are equivalent;
- Recognise factors of algebraic terms involving single brackets;
- Factorise algebraic expressions by taking out common factors.


## NOTES

Provide students with lots of practice.
This topic lends itself to regular reinforcement through starters in lessons.

## 2c. Expressions and substitution into formulae

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Write expressions to solve problems representing a situation;
- Substitute numbers in simple algebraic expressions;
- Substitute numbers into expressions involving brackets and powers;
- Substitute positive and negative numbers into expressions;
- Derive a simple formula, including those with squares, cubes and roots;
- Substitute numbers into a word formula;
- Substitute numbers into a formula.


## NOTES

Use formulae from mathematics and other subjects, expressed initially in words and then using letters and symbols.
Include substitution into the kinematics formulae given on the formula sheet, i.e. $v=u+a t$, $v^{2}-u^{2}=2 a s$, and $s=u t+\frac{1}{2} a t^{2}$.

UNIT 3: Drawing and interpreting graphs, tables and charts
Return to Overview

## KEYWORDS

Mean, median, mode, range, average, discrete, continuous, qualitative, quantitative, data, scatter graph, line of best fit, correlation, positive, negative, sample, population, stem and leaf, frequency, table, sort, pie chart, estimate

## 3a. Tables

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Use suitable data collection techniques (data to be integer and decimal values);
- Design and use data-collection sheets for grouped, discrete and continuous data, use inequalities for grouped data, and introduce $\leq$ and $\geq$ signs;
- Interpret and discuss the data;
- Sort, classify and tabulate data, both discrete and continuous quantitative data, and qualitative data;
- Construct tables for time-series data;
- Extract data from lists and tables;
- Use correct notation for time, 12- and 24-hour clock;
- Work out time taken for a journey from a timetable;
- Design and use two-way tables for discrete and grouped data;
- Use information provided to complete a two-way table;
- Calculate the total frequency from a frequency table;
- Read off frequency values from a table;
- Read off frequency values from a frequency table;
- Find greatest and least values from a frequency table;
- Identify the mode from a frequency table;
- Identify the modal class from a grouped frequency table.


## NOTES

Other averages are covered in unit 5, but you may choose to cover them in this unit.
Ensure that students are given the opportunity to draw and complete two-way tables from words.

## 3b. Charts and graphs

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Plotting coordinates in first quadrant and read graph scales in multiples;
- Produce:
- pictograms;
- composite bar charts;
- dual/comparative bar charts for categorical and ungrouped discrete data;
- bar-line charts;
- vertical line charts;
- line graphs;
- line graphs for time-series data;
- histograms with equal class intervals;
- stem and leaf (including back-to-back);
- Interpret data shown in
- pictograms;
- composite bar charts;
- dual/comparative bar charts;
- line graphs;
- line graphs for time-series data;
- histograms with equal class intervals;
- stem and leaf;
- Calculate total population from a bar chart or table;
- Find greatest and least values from a bar chart or table;
- Find the mode from a stem and leaf diagram;
- Identify the mode from a bar chart;
- Recognise simple patterns, characteristics, relationships in bar charts and line graphs.


## NOTES

Ensure that you include a variety of scales, including decimal numbers of millions and thousands, time scales in hours, minutes, seconds.

## 3c. Pie charts

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Draw circles and arcs to a given radius;
- Know there are 360 degrees in a full turn, 180 degrees in a half turn, and 90 degrees in a quarter turn;
- Measure and draw angles, to the nearest degree;
- Interpret tables; represent data in tables and charts;
- Know which charts to use for different types of data sets;
- Construct pie charts for categorical data and discrete/continuous numerical data;
- Interpret simple pie charts using simple fractions and percentages; $\frac{1}{2}, \frac{1}{4}$ and multiples of 10\% sections;
- From a pie chart:
- find the mode;
- find the total frequency;
- Understand that the frequency represented by corresponding sectors in two pie charts is dependent upon the total populations represented by each of the pie charts.


## NOTES

Relate $\frac{1}{4}, \frac{1}{2}$, etc to percentages.
Practise dividing by 20, 30, 40, 60, etc.
Compare pie charts to identify similarities and differences.
Angles when drawing pie charts should be accurate to $2^{\circ}$.

3d. Scatter graphs
Teaching time
(S4, S6)

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Draw scatter graphs;
- Interpret points on a scatter graph;
- Identify outliers and ignore them on scatter graphs;
- Draw the line of best fit on a scatter diagram by eye, and understand what it represents;
- Use the line of best fit make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing;
- Distinguish between positive, negative and no correlation using lines of best fit;
- Use a line of best fit to predict values of a variable given values of the other variable;
- Interpret scatter graphs in terms of the relationship between two variables;
- Interpret correlation in terms of the problem;
- Understand that correlation does not imply causality;
- State how reliable their predictions are, i.e. not reliable if extrapolated.


## NOTES

Students need to be constantly reminded of the importance of drawing a line of best fit.
Support with copy and complete statements, e.g. as the $\qquad$ increases, the $\qquad$ decreases. Statistically the line of best fit should pass through the coordinate representing the mean of the data.
Students should label the axes clearly, and use a ruler for all straight lines and a pencil for all drawing.
Remind students that the line of best fit does not necessarily go through the origin of the graph.

## KEYWORDS

Decimal, percentage, inverse, addition, subtraction, multiplication, division, fractions, mixed, improper, recurring, integer, decimal, terminating, percentage, VAT, increase, decrease, multiplier, profit, loss

| 4a. Fractions | Teaching time |
| :--- | ---: |
| $(N 1, N 2, N 3, N 12, R 3, S 2)$ | $6-8$ hours |

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Use diagrams to find equivalent fractions or compare fractions;
- Write fractions to describe shaded parts of diagrams;
- Express a given number as a fraction of another, using very simple numbers, some cancelling, and where the fraction is both $<1$ and $>1$;
- Write a fraction in its simplest form and find equivalent fractions;
- Order fractions, by using a common denominator;
- Compare fractions, use inequality signs, compare unit fractions;
- Convert between mixed numbers and improper fractions;
- Add and subtract fractions;
- Add fractions and write the answer as a mixed number;
- Multiply and divide an integer by a fraction;
- Multiply and divide a fraction by an integer, including finding fractions of quantities or measurements, and apply this by finding the size of each category from a pie chart using fractions;
- Understand and use unit fractions as multiplicative inverses;
- Multiply fractions: simplify calculations by cancelling first;
- Divide a fraction by a whole number;
- Divide fractions by fractions.


## NOTES

When expressing a given number as a fraction of another, start with very simple numbers $<1$, and include some cancelling before fractions using numbers $>1$.
When adding and subtracting fractions, start with same denominator, then where one denominator is a multiple of the other (answers $\leq 1$ ), and finally where both denominators have to be changed (answers $\leq 1$ ).
Regular revision of fractions is essential.
Demonstrate how to the use the fraction button on the calculator.
Use real-life examples where possible.
Use long division to illustrate recurring decimals.

## 4b. Fractions, decimals and percentages

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Recall the fraction-to-decimal conversion;
- Convert between fractions and decimals;
- Convert a fraction to a decimal to make a calculation easier, e.g. $0.25 \times 8=\frac{1}{4} \times 8$, or $\frac{3}{8} \times 10=0.375 \times 10 ;$
- Recognise recurring decimals and convert fractions such as $\frac{3}{7}, \frac{1}{3}$ and $\frac{2}{3}$ into recurring decimals;
- Compare and order fractions, decimals and integers, using inequality signs;
- Understand that a percentage is a fraction in hundredths;
- Express a given number as a percentage of another number;
- Convert between fractions, decimals and percentages;
- Order fractions, decimals and percentages, including use of inequality signs.


## NOTES

Students should be reminded of basic percentages and fraction conversions.
Emphasise the importance of being able to convert between fractions, decimals and percentages to make calculations easier.

## 4c. Percentages

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Express a given number as a percentage of another number;
- Find a percentage of a quantity without a calculator: $50 \%, 25 \%$ and multiples of $10 \%$ and 5\%;
- Find a percentage of a quantity or measurement (use measurements they should know from Key Stage 3 only);
- Calculate amount of increase/decrease;
- Use percentages to solve problems, including comparisons of two quantities using percentages;
- Percentages over $100 \%$;
- Use percentages in real-life situations, including percentages greater than 100\%:
- Price after VAT (not price before VAT);
- Value of profit or loss;
- Simple interest;
- Income tax calculations;
- Use decimals to find quantities;
- Find a percentage of a quantity, including using a multiplier;
- Use a multiplier to increase or decrease by a percentage in any scenario where percentages are used;
- Understand the multiplicative nature of percentages as operators.


## NOTES

When finding a percentage of a quantity or measurement, use only measurements they should know from Key Stage 3.
Amounts of money should always be rounded to the nearest penny.
Use real-life examples where possible.
Emphasise the importance of being able to convert between decimals and percentages and the use of decimal multipliers to make calculations easier.

UNIT 5: Equations, inequalities and sequences

Return to Overview

## KEYWORDS

Arithmetic, geometric, function, sequence, $n$th term, derive, quadratic, triangular, cube, square, odd, even, solve, change, subject, inequality, represent, substitute, bracket, expand, linear, equation, balance, accuracy

## 5a. Equations

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Select an expression/equation/formula/identity from a list;
- Write expressions and set up simple equations;
- Use function machines;
- Solve simple equations;
- Solve linear equations, with integer coefficients, in which the unknown appears on either side or on both sides of the equation;
- Solve linear equations which contain brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution;
- Solve linear equations in one unknown, with integer or fractional coefficients;
- Rearrange simple equations;
- Substitute into a formula, and solve the resulting equation;
- Find an approximate solution to a linear equation using a graph;
- Solve angle or perimeter problems using algebra.
- Write an equation to solve a word problem.


## NOTES

Emphasise good use of notation.
Students need to realise that not all linear equations can be solved by observation or trial and improvement, and hence the use of a formal method is important.
Students can leave their answer in fraction form where appropriate.

## 5b. Inequalities

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Show inequalities on number lines;
- Write down whole number values that satisfy an inequality;
- Solve an inequality such as $-3<2 x+1<7$ and show the solution set on a number line;
- Solve two inequalities in $x$, find the solution sets and compare them to see which value of $x$ satisfies both;
- Use the correct notation to show inclusive and exclusive inequalities;
- Construct inequalities to represent a set shown on a number line;
- Solve simple linear inequalities in one variable, and represent the solution set on a number line;
- Round answers to a given degree of accuracy.


## NOTES

Emphasise the importance of leaving their answer as an inequality (and not change to =).

## 5c. Sequences

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Recognise sequences of odd and even numbers, and other sequences including Fibonacci sequences;
- Use function machines to find terms of a sequence;
- Write the term-to-term definition of a sequence in words;
- Find a specific term in the sequence using position-to-term or term-to-term rules;
- Generate arithmetic sequences of numbers, triangular number, square and cube integers and sequences derived from diagrams;
- Recognise such sequences from diagrams and draw the next term in a pattern sequence;
- Find the next term in a sequence, including negative values;
- Find the $n$th term for a pattern sequence;
- Find the $n$th term of a linear sequence;
- Find the $n$th term of an arithmetic sequence;
- Use the $n$th term of an arithmetic sequence to generate terms;
- Use the $n$th term of an arithmetic sequence to decide if a given number is a term in the sequence, or find the first term over a certain number;
- Use the $n$th term of an arithmetic sequence to find the first term greater/less than a certain number;
- Continue a geometric progression and find the term-to-term rule, including negatives, fraction and decimal terms;
- Continue a quadratic sequence and use the $n$th term to generate terms;
- Distinguish between arithmetic and geometric sequences.


## NOTES

Emphasise use of $3 n$ meaning $3 \times n$.

Students need to be clear on the description of the pattern in words, the difference between the terms and the algebraic description of the $n$th term.
Students are not expected to find the $n$th term of a quadratic sequence.
UNIT 6: Angles, polygons and parallel lines

Return to Overview

## KEYWORDS

Quadrilateral, angle, polygon, interior, exterior, proof, tessellation, rotational symmetry, parallel, corresponding, alternate, co-interior, vertices, edge, face, sides, triangle, perpendicular, isosceles, scalene, clockwise, anticlockwise, hexagons, heptagons, octagons, decagons, obtuse, acute, reflex, quadrilateral, triangle, regular, irregular, two-dimensional, three-dimensional, measure, line, angle, order, intersecting

## 6a. Properties of shapes, parallel lines and angle facts

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Estimate sizes of angles;
- Measure angles using a protractor;
- Use geometric language appropriately;
- Use letters to identify points, lines and angles;
- Use two-letter notation for a line and three-letter notation for an angle;
- Describe angles as turns and in degrees;
- Understand clockwise and anticlockwise;
- Know that there are $360^{\circ}$ in a full turn, $180^{\circ}$ in a half turn and $90^{\circ}$ in a quarter turn;
- Identify a line perpendicular to a given line;
- Mark perpendicular lines on a diagram and use their properties;
- Identify parallel lines;
- Mark parallel lines on a diagram and use their properties;
- Recall the properties and definitions of special types of quadrilaterals, including symmetry properties;
- List the properties of each special type of quadrilateral, or identify (name) a given shape;
- Draw sketches of shapes;
- Name all quadrilaterals that have a specific property;
- Identify quadrilaterals from everyday usage;
- Given some information about a shape on coordinate axes, complete the shape;
- Classify quadrilaterals by their geometric properties;
- Understand and use the angle properties of quadrilaterals;
- Use the fact that angle sum of a quadrilateral is $360^{\circ}$;
- Use geometrical language appropriately and give reasons for angle calculations;
- Recall and use properties of angles at a point, angles at a point on a straight line, right angles, and vertically opposite angles;
- Distinguish between scalene, equilateral, isosceles and right-angled triangles;
- Derive and use the sum of angles in a triangle;
- Find a missing angle in a triangle, using the angle sum of a triangle is $180^{\circ}$;
- Understand and use the angle properties of triangles, use the symmetry property of isosceles triangle to show that base angles are equal;
- Use the side/angle properties of isosceles and equilateral triangles;
- Show step-by-step deduction when solving problems;
- Understand and use the angle properties of intersecting lines;
- Understand a proof that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices;
- Find missing angles using properties of corresponding and alternate angles;
- Understand and use the angle properties of parallel lines.


## NOTES

Emphasise that diagrams in examinations are seldom drawn accurately. Make sure drawings are neat, labelled and accurate.
Give students lots of practice.
Angles should be accurate to within $2^{\circ}$.
Investigate Rangoli patterns.
Use tracing paper to assist with symmetry questions.
Ask students to find their own examples of symmetry in real life.
Emphasise that diagrams in examinations are seldom drawn accurately.
Make sure drawings are neat, labelled and accurate.
Students should have plenty of practice drawing examples to illustrate the properties and encourage them to check their drawings.
Emphasise the need to give geometric reasons when required.

## 6b. Interior and exterior angles of polygons

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Recognise and name pentagons, hexagons, heptagons, octagons and decagons;
- Understand 'regular' and 'irregular' as applied to polygons;
- Use the sum of angles of irregular polygons;
- Calculate and use the sums of the interior angles of polygons;
- Calculate and use the angles of regular polygons;
- Use the sum of the interior angles of an $n$-sided polygon;
- Use the sum of the exterior angles of any polygon is $360^{\circ}$;
- Use the sum of the interior angle and the exterior angle is $180^{\circ}$;
- Identify shapes which are congruent (by eye);
- Explain why some polygons fit together and others do not;


## NOTES

Use examples of tiling patterns with simple shapes to help students investigate if shapes 'fit together'.

UNIT 7: Averages and range, sampling, collecting data, analysing data

Return to Overview

## KEYWORDS

Mean, median, mode, range, average, discrete, continuous, qualitative, quantitative, data, sample, population, stem and leaf, frequency, table, sort, pie chart, estimate, primary, secondary, interval, midpoint, survey

## 7a. Statistics and sampling

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Specify the problem and:
- plan an investigation;
- decide what data to collect and what statistical analysis is needed;
- consider fairness;
- Recognise types of data: primary secondary, quantitative and qualitative;
- Identify which primary data they need to collect and in what format, including grouped data;
- Collect data from a variety of suitable primary and secondary sources;
- Understand how sources of data may be biased;
- Explain why a sample may not be representative of a whole population;
- Understand sample and population.


## NOTES

Emphasise the difference between primary and secondary sources and remind students about the different between discrete and continuous data.
Discuss sample size and mention that a census is the whole population (the UK census takes place every 10 years in a year ending with a 1 - the next one is due in 2021).
Specify the problem and planning for data collection is not included in the programme of study but is a perquisite to understand the context of the topic.
Writing a questionnaire is not part of the new specification, but is a good topic to demonstrate bias and ways to reduce bias in terms of timing, location and question types that can introduce bias.

## 7b. The averages

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Calculate the mean, mode, median and range for discrete data;
- Can interpret and find a range of averages as follows:
- median, mean and range from a (discrete) frequency table;
- range, modal class, interval containing the median, and estimate of the mean from a grouped data frequency table;
- mode and range from a bar chart;
- median, mode and range from stem and leaf diagrams;
- mean from a bar chart;
- Understand that the expression 'estimate' will be used where appropriate, when finding the mean of grouped data using mid-interval values;
- Compare the mean, median, mode and range (as appropriate) of two distributions using bar charts, dual bar charts, pictograms and back-to-back stem and leaf;
- Recognise the advantages and disadvantages between measures of average.


## NOTES

Encourage students to cross out the midpoints of each group once they have used these numbers to in $m \times f$. This helps students to avoid summing $m$ instead of $f$.
Remind students how to find the midpoint of two numbers.
Emphasise that continuous data is measured, i.e. length, weight, and discrete data can be counted, i.e. number of shoes.
When comparing the mean and range of two distributions support with 'copy and complete' sentences, or suggested wording.

UNIT 8: Perimeter, area and volume I

Return to Overview

## KEYWORDS

Triangle, rectangle, parallelogram, trapezium, area, perimeter, formula, length, width, prism, compound, measurement, polygon, cuboid, volume, symmetry, vertices, edge, face, units, conversion

## 8a. Perimeter and area

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Indicate given values on a scale, including decimal value;
- Know that measurements using real numbers depend upon the choice of unit;
- Convert between units of measure within one system, including time;
- Convert metric units to metric units;
- Make sensible estimates of a range of measures in everyday settings;
- Measure shapes to find perimeters and areas using a range of scales;
- Find the perimeter of rectangles and triangles;
- Find the perimeter of parallelograms and trapezia;
- Find the perimeter of compound shapes;
- Recall and use the formulae for the area of a triangle and rectangle;
- Find the area of a rectangle and triangle;
- Find the area of a trapezium and recall the formula;
- Find the area of a parallelogram;
- Calculate areas and perimeters of compound shapes made from triangles and rectangles;
- Estimate surface areas by rounding measurements to 1 significant figure;
- Find the surface area of a prism;
- Find surface area using rectangles and triangles;
- Convert between metric area measures.


## NOTES

Use questions that involve different metric measures that need converting.
Measurement is essentially a practical activity: use a range of everyday shapes to bring reality to lessons.
Ensure that students are clear about the difference between perimeter and area.
Practical examples help to clarify the concepts, i.e. floor tiles, skirting board, etc.

## 8b. 3D forms and volume

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Identify and name common solids: cube, cuboid, cylinder, prism, pyramid, sphere and cone;
- Sketch nets of cuboids and prisms;
- Recall and use the formula for the volume of a cuboid;
- Find the volume of a prism, including a triangular prism, cube and cuboid;
- Calculate volumes of right prisms and shapes made from cubes and cuboids;
- Estimate volumes etc by rounding measurements to 1 significant figure;
- Convert between metric volume measures;
- Convert between metric measures of volume and capacity e.g. $1 \mathrm{ml}=1 \mathrm{~cm}^{3}$.


## NOTES

Discuss the correct use of units.
Drawings should be done in pencil.
Consider 'how many small boxes fit in a larger box'type questions.
Practical examples should be used to enable students to understand the difference between perimeter, area and volume.

UNIT 9: Real-life and algebraic linear graphs

## KEYWORDS

Linear, graph, distance, time, coordinate, quadrant, real-life graph, gradient, intercept, function, solution, parallel

## 9a. Real-life graphs

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Use input/output diagrams;
- Use axes and coordinates to specify points in all four quadrants in 2D;
- Identify points with given coordinates and coordinates of a given point in all four quadrants;
- Find the coordinates of points identified by geometrical information in 2D (all four quadrants);
- Find the coordinates of the midpoint of a line segment;
- Draw, label and scale axes;
- Read values from straight-line graphs for real-life situations;
- Draw straight line graphs for real-life situations, including ready reckoner graphs, conversion graphs, fuel bills graphs, fixed charge and cost per unit;
- Draw distance-time graphs and velocity-time graphs;
- Work out time intervals for graph scales;
- Interpret distance-time graphs, and calculate: the speed of individual sections, total distance and total time;
- Interpret information presented in a range of linear and non-linear graphs;
- Interpret graphs with negative values on axes;
- Interpret gradient as the rate of change in distance-time and speed-time graphs, graphs of containers filling and emptying, and unit price graphs.


## NOTES

Clear presentation of axes is important.
Ensure that you include questions that include axes with negative values to represent, for example, time before present time, temperature or depth below sea level.
Careful annotation should be encouraged: it is good practice to get the students to check that they understand the increments on the axes.
Use standard units of measurement to draw conversion graphs.
Use various measures in distance-time and velocity-time graphs, including miles, kilometres, seconds, and hours.

## 9b. Straight-line graphs

## OBJECTIVES

- By the end of the sub-unit, students should be able to:
- Use function machines to find coordinates (i.e. given the input $x$, find the output $y$ );
- Plot and draw graphs of $y=a, x=a, y=x$ and $y=-x$;
- Recognise straight-line graphs parallel to the axes;
- Recognise that equations of the form $y=m x+c$ correspond to straight-line graphs in the coordinate plane;
- Plot and draw graphs of straight lines of the form $y=m x+c$ using a table of values;
- Sketch a graph of a linear function, using the gradient and $y$-intercept;
- Identify and interpret gradient from an equation $y=m x+c$;
- Identify parallel lines from their equations;
- Plot and draw graphs of straight lines in the form $a x+b y=c$;
- Find the equation of a straight line from a graph;
- Find the equation of the line through one point with a given gradient;
- Find approximate solutions to a linear equation from a graph;
- Find the gradient of a straight line from real-life graphs too.


## NOTES

Emphasise the importance of drawing a table of values when not given one. Values for a table should be taken from the $x$-axis.

UNIT 10: Transformations

## KEYWORDS

Transformation, rotation, reflection, enlargement, translation, single, combination, scale factor, mirror line, centre of rotation, centre of enlargement, column vector, vector, similarity, congruent, angle, direction, coordinate, describe

## 10a. Transformations I: rotations and translations

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Identify congruent shapes by eye;
- Understand clockwise and anticlockwise;
- Understand that rotations are specified by a centre, an angle and a direction of rotation;
- Find the centre of rotation, angle and direction of rotation and describe rotations;
- Describe a rotation fully using the angle, direction of turn, and centre;
- Rotate a shape about the origin or any other point on a coordinate grid;
- Draw the position of a shape after rotation about a centre (not on a coordinate grid);
- Identify correct rotations from a choice of diagrams;
- Understand that translations are specified by a distance and direction using a vector;
- Translate a given shape by a vector;
- Describe and transform 2D shapes using single translations on a coordinate grid;
- Use column vectors to describe translations;
- Understand that distances and angles are preserved under rotations and translations, so that any figure is congruent under either of these transformations.


## NOTES

Emphasise the need to describe the transformations fully, and if asked to describe a 'single' transformation they should not include two types.
Include rotations with the centre of rotation inside the shape.
Use trial and error with tracing paper to find the centre of rotation.
It is essential that the students check the increments on the coordinate grid when translating shapes.

## 10b. Transformations II: reflections and enlargements

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Understand that reflections are specified by a mirror line;
- Identify correct reflections from a choice of diagrams;
- Understand that reflections are specified by a mirror line;
- Identify the equation of a line of symmetry;
- Transform 2D shapes using single reflections (including those not on coordinate grids) with vertical, horizontal and diagonal mirror lines;
- Describe reflections on a coordinate grid;
- Scale a shape on a grid (without a centre specified);
- Understand that an enlargement is specified by a centre and a scale factor;
- Enlarge a given shape using $(0,0)$ as the centre of enlargement, and enlarge shapes with a centre other than ( 0,0 );
- Find the centre of enlargement by drawing;
- Describe and transform 2D shapes using enlargements by:
- a positive integer scale factor;
- a fractional scale factor;
- Identify the scale factor of an enlargement of a shape as the ratio of the lengths of two corresponding sides, simple integer scale factors, or simple fractions;
- Understand that distances and angles are preserved under reflections, so that any figure is congruent under this transformation;
- Understand that similar shapes are enlargements of each other and angles are preserved define similar in this unit;
- Describe and transform 2D shapes using combined rotations, reflections, translations, or enlargements.


## NOTES

Emphasise the need to describe the transformations fully and if asked to describe a 'single' transformation they should not include two types.
Students may need reminding about how to find the equations of straight lines, including those parallel to the axes.
When reflecting shapes, the students must include mirror lines on or through original shapes.
As an extension, consider reflections with the mirror line through the shape and enlargements with the centre of enlargement inside the shape.
NB enlargement using negative scale factors is not included.

## KEYWORDS

Ratio, proportion, share, parts, fraction, function, direct proportion, inverse proportion, graphical, linear, compare

## 11a. Ratio

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Understand and express the division of a quantity into a of number parts as a ratio;
- Write ratios in their simplest form;
- Write/interpret a ratio to describe a situation;
- Share a quantity in a given ratio including three-part ratios;
- Solve a ratio problem in context:
- use a ratio to find one quantity when the other is known;
- use a ratio to compare a scale model to a real-life object;
- use a ratio to convert between measures and currencies;
- problems involving mixing, e.g. paint colours, cement and drawn conclusions;
- Compare ratios;
- Write ratios in form $1: m$ or $m: 1$;
- Write a ratio as a fraction;
- Write a ratio as a linear function;
- Write lengths, areas and volumes of two shapes as ratios in simplest form;
- Express a multiplicative relationship between two quantities as a ratio or a fraction.


## NOTES

Emphasise the importance of reading the question carefully.
Include ratios with decimals 0.2 : 1 .
Converting imperial units to imperial units aren't specifically in the programme of study, but still useful and provide a good context for multiplicative reasoning.
It is also useful generally for students to know rough metric equivalents of commonly used imperial measures, such as pounds, feet, miles and pints.

## 11b. Proportion

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Understand and use proportion as equality of ratios;
- Solve word problems involving direct and indirect proportion;
- Work out which product is the better buy;
- Scale up recipes;
- Convert between currencies;
- Find amounts for 3 people when amount for 1 given;
- Solve proportion problems using the unitary method;
- Recognise when values are in direct proportion by reference to the graph form;
- Understand inverse proportion: as $x$ increases, $y$ decreases (inverse graphs done in later unit);
- Recognise when values are in direct proportion by reference to the graph form;
- Understand direct proportion ---> relationship $y=k x$.


## NOTES

Find out/prove whether two variables are in direct proportion by plotting the graph and using it as a model to read off other values.
Possible link with scatter graphs.

## UNIT 12: Right-angled triangles: Pythagoras and Trigonometry

## KEYWORDS

Triangle, right angle, angle, Pythagoras' Theorem, sine, cosine, tan, trigonometry, opposite, hypotenuse, adjacent, ratio, elevation, depression, length, accuracy

## OBJECTIVES

By the end of the unit, students should be able to:

- Understand, recall and use Pythagoras' Theorem in 2D, including leaving answers in surd form;
- Given 3 sides of a triangle, justify if it is right-angled or not;
- Calculate the length of the hypotenuse in a right-angled triangle, including decimal lengths and a range of units;
- Find the length of a shorter side in a right-angled triangle;
- Apply Pythagoras' Theorem with a triangle drawn on a coordinate grid;
- Calculate the length of a line segment $A B$ given pairs of points;
- Understand, use and recall the trigonometric ratios sine, cosine and tan, and apply them to find angles and lengths in general triangles in 2D figures;
- Use the trigonometric ratios to solve 2D problems;
- Find angles of elevation and depression;
- Round answers to appropriate degree of accuracy, either to a given number of significant figures or decimal places, or make a sensible decision on rounding in context of question;
- Know the exact values of $\sin \theta$ and $\cos \theta$ for $\theta=0^{\circ}, 30^{\circ}, 45^{\circ}, 60^{\circ}$ and $90^{\circ}$; know the exact value of $\tan \theta$ for $\theta=0^{\circ}, 30^{\circ}, 45^{\circ}$ and $60^{\circ}$.


## NOTES

Students may need reminding about surds.
Drawing the squares on the 3 sides will help to illustrate the theorem.
Include examples with triangles drawn in all four quadrants.
Scale drawings are not acceptable.
Calculators need to be in degree mode.
To find in right-angled triangles the exact values of $\sin \theta$ and $\cos \theta$ for $\theta=0^{\circ}, 30^{\circ}, 45^{\circ}, 60^{\circ}$ and $90^{\circ}$, use triangles with angles of $30^{\circ}, 45^{\circ}$ and $60^{\circ}$.
Use a suitable mnemonic to remember SOHCAHTOA.
Use Pythagoras' Theorem and trigonometry together.

UNIT 13: Probability

## KEYWORDS

Probability, dependent, independent, conditional, tree diagrams, sample space, outcomes, theoretical, relative frequency, fairness, experimental

## 13a. Probability I

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Distinguish between events which are impossible, unlikely, even chance, likely, and certain to occur;
- Mark events and/or probabilities on a probability scale of 0 to 1 ;
- Write probabilities in words or fractions, decimals and percentages;
- Find the probability of an event happening using theoretical probability;
- Use theoretical models to include outcomes using dice, spinners, coins;
- List all outcomes for single events systematically;
- Work out probabilities from frequency tables;
- Work out probabilities from two-way tables;
- Record outcomes of probability experiments in tables;
- Add simple probabilities;
- Identify different mutually exclusive outcomes and know that the sum of the probabilities of all outcomes is 1 ;
- Using $1-p$ as the probability of an event not occurring where $p$ is the probability of the event occurring;
- Find a missing probability from a list or table including algebraic terms.


## NOTES

Use this as an opportunity for practical work.
Probabilities written in fraction form should be cancelled to their simplest form.

## 13b. Probability II

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Find the probability of an event happening using relative frequency;
- Estimate the number of times an event will occur, given the probability and the number of trials - for both experimental and theoretical probabilities;
- List all outcomes for combined events systematically;
- Use and draw sample space diagrams;
- Work out probabilities from Venn diagrams to represent real-life situations and also 'abstract' sets of numbers/values;
- Use union and intersection notation;
- Compare experimental data and theoretical probabilities;
- Compare relative frequencies from samples of different sizes;
- Find the probability of successive events, such as several throws of a single dice;
- Use tree diagrams to calculate the probability of two independent events;
- Use tree diagrams to calculate the probability of two dependent events.


## NOTES

Probability without replacement is best illustrated visually and by initially working out probability 'with' replacement.
Encourage students to work 'across' the branches working out the probability of each successive event. The probability of the combinations of outcomes should $=1$.
Emphasise that were an experiment repeated it will usually lead to different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics.
Probabilities written in fraction form should be cancelled to their simplest form.

## UNIT 14: Multiplicative reasoning: more percentages, rates of change, compound measures

## KEYWORDS

Ratio, proportion, best value, proportional change, compound measure, density, mass, volume, speed, distance, time, density, mass, volume, pressure, acceleration, velocity, inverse, direct

## OBJECTIVES

By the end of the unit, students should be able to:

- Understand and use compound measures:
- density;
- pressure;
- speed:
- convert between metric speed measures;
- read values in $\mathrm{km} / \mathrm{h}$ and mph from a speedometer;
- calculate average speed, distance, time - in miles per hour as well as metric measures;
- use kinematics formulae from the formulae sheet to calculate speed, acceleration (with variables defined in the question);
- change $\mathrm{d} / \mathrm{t}$ in $\mathrm{m} / \mathrm{s}$ to a formula in $\mathrm{km} / \mathrm{h}$, i.e. $\mathrm{d} / \mathrm{t} \times(60 \times 60) / 1000-$ with support;
- Express a given number as a percentage of another number in more complex situations;
- Calculate percentage profit or loss;
- Make calculations involving repeated percentage change, not using the formula;
- Find the original amount given the final amount after a percentage increase or decrease;
- Use compound interest;
- Use a variety of measures in ratio and proportion problems:
- currency conversion;
- rates of pay;
- best value;
- Set up, solve and interpret the answers in growth and decay problems;
- Understand that $X$ is inversely proportional to $Y$ is equivalent to $X$ is proportional to $\frac{1}{Y}$;
- Interpret equations that describe direct and inverse proportion.


## NOTES

Encourage students to use a single multiplier.
Include simple fractional percentages of amounts with compound interest and encourage use of single multipliers.
Amounts of money should be rounded to the nearest penny, but emphasise the importance of not rounding until the end of the calculation if doing in stages.
Use a formula triangle to help students see the relationship for compound measures - this will help them evaluate which inverse operations to use.
Help students to recognise the problem they are trying to solve by the unit measurement given, e.g. $\mathrm{km} / \mathrm{h}$ is a unit of speed as it is speed divided by a time.

## UNIT 15: Constructions: triangles, nets, plan and elevation, loci, scale drawings and bearings

## KEYWORDS

Construct, circle, arc, sector, face, edge, vertex, two-dimensional, three-dimensional, solid, elevations, congruent, angles, regular, irregular, bearing, degree, bisect, perpendicular, loci, map, scale, plan, region

## 15a. Plans and elevations

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Understand clockwise and anticlockwise;
- Draw circles and arcs to a given radius or given the diameter;
- Measure and draw lines, to the nearest mm;
- Measure and draw angles, to the nearest degree;
- Know and use compass directions;
- Draw sketches of 3D solids;
- Know the terms face, edge and vertex;
- Identify and sketch planes of symmetry of 3D solids;
- Use isometric grids to draw 2D representations of 3D solids;
- Make accurate drawings of triangles and other 2D shapes using a ruler and a protractor;
- Construct diagrams of everyday 2D situations involving rectangles, triangles, perpendicular and parallel lines;
- Understand and draw front and side elevations and plans of shapes made from simple solids;
- Given the front and side elevations and the plan of a solid, draw a sketch of the 3D solid.


## NOTES

This is a very practical topic, and provides opportunities for some hands-on activities.
Whilst not an explicit objective, it is useful for students to draw and construct nets and show how they fold to make 3D solids, allowing students to make the link between 3D shapes and their nets. This will enable students to understand that there is often more than one net that can form a 3D shape.

## 15b. Constructions, loci and bearings

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Understand congruence, as two shapes that are the same size and shape;
- Visually identify shapes which are congruent;
- Use straight edge and a pair of compasses to do standard constructions:
- understand, from the experience of constructing them, that triangles satisfying SSS, SAS, ASA and RHS are unique, but SSA triangles are not;
- construct the perpendicular bisector of a given line;
- construct the perpendicular from a point to a line;
- construct the bisector of a given angle;
- construct angles of $90^{\circ}, 45^{\circ}$;
- Draw and construct diagrams from given instructions, including the following:
- a region bounded by a circle and an intersecting line;
- a given distance from a point and a given distance from a line;
- equal distances from two points or two line segments;
- regions may be defined by 'nearer to' or 'greater than';
- Find and describe regions satisfying a combination of loci;
- Use constructions to solve loci problems (2D only);
- Use and interpret maps and scale drawings;
- Estimate lengths using a scale diagram;
- Make an accurate scale drawing from a diagram;
- Use three-figure bearings to specify direction;
- Mark on a diagram the position of point $B$ given its bearing from point $A$;
- Give a bearing between the points on a map or scaled plan;
- Given the bearing of a point $A$ from point $B$, work out the bearing of $B$ from $A$;
- Use accurate drawing to solve bearings problems;
- Solve locus problems including bearings.


## NOTES

Drawings should be done in pencil.
Relate loci problems to real-life scenarios, including mobile phone masts and coverage. Construction lines should not be erased.

UNIT 16: Algebra: quadratic equations and graphs

## KEYWORDS

Quadratic, function, solve, expand, factorise, simplify, expression, graph, curve, factor, coefficient, bracket

## 16a. Quadratic equations: expanding and factorising

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Define a 'quadratic' expression;
- Multiply together two algebraic expressions with brackets;
- Square a linear expression, e.g. $(x+1)^{2}$;
- Factorise quadratic expressions of the form $x^{2}+b x+c$;
- Factorise a quadratic expression $x^{2}-a^{2}$ using the difference of two squares;
- Solve quadratic equations by factorising;
- Find the roots of a quadratic function algebraically.


## NOTES

This unit can be extended by including quadratics where $a \neq 1$.
Emphasise the fact that $x^{2}$ and $x$ are different 'types' of term - illustrate this with numbers.

## 16b. Quadratic equations: graphs

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Generate points and plot graphs of simple quadratic functions, then more general quadratic functions;
- Identify the line of symmetry of a quadratic graph;
- Find approximate solutions to quadratic equations using a graph;
- Interpret graphs of quadratic functions from real-life problems;
- Identify and interpret roots, intercepts and turning points of quadratic graphs.


## NOTES

The graphs should be drawn freehand and in pencil, joining points using a smooth curve.
Encourage efficient use of the calculator.
Extension work can be through plotting cubic and reciprocal graphs, solving simultaneous equations graphically.

## UNIT 17: Perimeter, area and volume 2: circles, cylinders, cones and spheres

## KEYWORDS

Area, perimeter, formula, length, width, measurement, volume, circle, segment, arc, sector, cylinder, circumference, radius, diameter, pi, sphere, cone, hemisphere, segment, accuracy, surface area

## OBJECTIVES

By the end of the unit, students should be able to:

- Recall the definition of a circle;
- Identify, name and draw parts of a circle including tangent, chord and segment;
- Recall and use formulae for the circumference of a circle and the area enclosed by a circle circumference of a circle $=2 \pi r=\pi d$, area of a circle $=\pi r^{2}$;
- Find circumferences and areas enclosed by circles;
- Use $\pi \approx 3.142$ or use the $\pi$ button on a calculator;
- Give an answer to a question involving the circumference or area of a circle in terms of $\pi$;
- Find radius or diameter, given area or perimeter of a circles;
- Find the perimeters and areas of semicircles and quarter-circles;
- Calculate perimeters and areas of composite shapes made from circles and parts of circles;
- Calculate arc lengths, angles and areas of sectors of circles;
- Find the surface area of a cylinder;
- Find the volume of a cylinder;
- Find the surface area and volume of spheres, pyramids, cones and composite solids;
- Round answers to a given degree of accuracy.


## NOTES

Emphasise the need to learn the circle formula: 'Cherry Pie's Delicious' and 'Apple Pies are too' are good ways to remember them.
Formulae for curved surface area and volume of a sphere, and surface area and volume of a cone, will be given on the formulae sheet in the examination.
Ensure that students know it is more accurate to leave answers in terms of $\pi$ but only when asked to do so.

## UNIT 18: More fractions, reciprocals, standard form, zero and negative indices

## KEYWORDS

Add, subtract, multiply, divide, mixed, improper, fraction, decimal, indices, standard form, power, reciprocal, index

## 18a. Fractions

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Add and subtract mixed number fractions;
- Multiply mixed number fractions;
- Divide mixed numbers by whole numbers and vice versa;
- Find the reciprocal of an integer, decimal or fraction;
- Understand 'reciprocal' as multiplicative inverse, knowing that any non-zero number multiplied by its reciprocal is 1 (and that zero has no reciprocal because division by zero is not defined).


## NOTES

Regular revision of fractions is essential.
Demonstrate how to the use the fraction button on the calculator.
Use real-life examples where possible.

## 18b. Indices and standard form

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer powers, fractions and powers of a power;
- Use numbers raised to the power zero, including the zero power of 10;
- Convert large and small numbers into standard form and vice versa;
- Add and subtract numbers in standard form;
- Multiply and divide numbers in standard form;
- Interpret a calculator display using standard form and know how to enter numbers in standard form.


## NOTES

Negative fractional indices are not included at Foundation tier, but you may wish to extend the work to include these.
Standard form is used in science and there are lots of cross curricular opportunities.
Students need to be provided with plenty of practice in using standard form with calculators.

## KEYWORDS

Vector, direction, magnitude, scalar, multiple, parallel, collinear, ratio, column vector, congruence, side, angle, compass, construction, shape, volume, length, area, volume, scale factor, enlargement, similar, perimeter,

## 19a. Similarity and congruence in 2D

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Use the basic congruence criteria for triangles (SSS, SAS, ASA and RHS);
- Solve angle problems involving congruence;
- Identify shapes which are similar; including all circles or all regular polygons with equal number of sides;
- Understand similarity of triangles and of other plane shapes, use this to make geometric inferences, and solve angle problems using similarity;
- Identify the scale factor of an enlargement of a shape as the ratio of the lengths of two corresponding sides;
- Understand the effect of enlargement on perimeter of shapes;
- Solve problems to find missing lengths in similar shapes;
- Know that scale diagrams, including bearings and maps are `similar' to the real-life examples.


## NOTES

Use simple scale factors that are easily calculated mentally to introduce similar shapes. Reinforce the fact that the sizes of angles are maintained when a shape is enlarged. Make links between similarity and trigonometric ratios.

## 19b. Vectors

## OBJECTIVES

By the end of the sub-unit, students should be able to:

- Understand and use column notation in relation to vectors;
- Be able to represent information graphically given column vectors;
- Identify two column vectors which are parallel;
- Calculate using column vectors, and represent graphically, the sum of two vectors, the difference of two vectors and a scalar multiple of a vector.


## NOTES

Students find manipulation of column vectors relatively easy compared to the pictorial and algebraic manipulation methods - encourage them to draw any vectors that they calculate on the picture.

## UNIT 20: Rearranging equations, graphs of cubic and <br> reciprocal functions and simultaneous equations

## KEYWORDS

Reciprocal, linear, gradient, functions, direct, indirect, estimate, cubic, subject, rearrange, simultaneous, substitution, elimination, proof

## OBJECTIVES

By the end of the unit, students should be able to:

- Know the difference between an equation and an identity and use and understand the $\neq$ symbol;
- Change the subject of a formula involving the use of square roots and squares;
- Answer 'show that' questions using consecutive integers $(n, n+1)$, squares $a^{2}, b^{2}$, even numbers $2 n$, and odd numbers $2 n+1$;
- Solve problems involving inverse proportion using graphs, and read values from graphs;
- Find the equation of the line through two given points;
- Recognise, sketch and interpret graphs of simple cubic functions;
- Recognise, sketch and interpret graphs of the reciprocal function $y=\frac{1}{x}$ with $x \neq 0$;
- Use graphical representations of indirect proportion to solve problems in context;
- identify and interpret the gradient from an equation $a x+b y=c$;
- Write simultaneous equations to represent a situation;
- Solve simultaneous equations (linear/linear) algebraically and graphically;
- Solve simultaneous equations representing a real-life situation, graphically and algebraically, and interpret the solution in the context of the problem;


## NOTES

Emphasise the need for good algebraic notation.

